Octahedron institute, chandan nagar

office no 2, 1st floor chandan complex

Class 12 - Mathematics

app of derivative

Maximum Marks: 100 **Time Allowed: 2 hours**

General Instructions:

ANSWER ALL QUESTIONS

Section A

1. Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$ **2**
2. Prove that the function f given by f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$. **2**
3. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value. **2**
4. Find the intervals in which the function given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is **2**
   a. strictly increasing
   b. strictly decreasing.
5. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area. **2**
6. Find the point at which the tangent to the curve y = x^3 - $3x^2$ -9x + 7 is parallel to the x - axis. **2**
7. Find the slope of tangent to the curve y = x^3 - x + 1$y = x^3 - x + 1$ at the given point whose x - coordinate is 2. **2**
8. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R. **2**
9. Find the approximate value of $(0.0037)^{\frac{1}{2}}$ **2**
10. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima. **2**
11. Find the interval in which the function given by is increaseasing or decreasing $f(x) = \sin 3x, x \in \left[0, \frac{\pi}{2}\right]$ **4**
12. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are (i) parallel to x – **4**

axis (ii) parallel to y – axis

13. A men of height 2 m walks at a uniform speed of 5 km/h away from a lamp post which is 6 m high. Find the rate at which the lengths of his shadow increase. **4**

14. Find the co-ordinates of the point on the curve $\sqrt{x}+\sqrt{y}=4$ at which tangent is equally inclined to the axes. **4**

15. Find the approximate value of f(5.001) where f(x) = x^3 - $7x^2$ + 15. **4**

16. Find the maximum area of an isosceles Δ inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis. **4**

17. Find the equation, of the tangent line to the curve $y=x^2-2x+7$ which is **4**
    a. Parallels to the line $2x-y+9=0$
    b. Perpendicular to the line $5y-15x=13$

18. Find the interval in which the function f given by $f(x)=x^3+\frac{1}{x^3}, x\neq 0$ is **4**
    i. increasing
    ii. decreasing.

19. For the curve $y=4x^3-2x^5$, find all the point at which the tangent passes through the origin. **4**

20. Find the intervals in which the function f given by $f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is **4**
    i. increasing
    ii. decreasing

21. A square piece of tin of side 18cm is to be made into a box without top by cutting a square from each corner and folding of the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible? **4**

22. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. **4**

23. Find the maximum area of an isosceles Δ inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis **4**

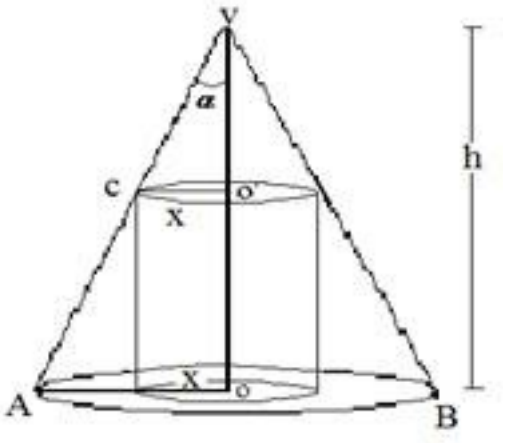
24. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3}+b^{3/2})$. **4**

25. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{2/3}+b^{2/3}\right)^{3/2}$ **4**

26. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made **4**

26. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined areas of the square and the circle is minimum. **4**

27. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi^3 h \tan\alpha$ **4**



28. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ Also find the maximum volume. **4**

29. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also, find the maximum volume. **4**

30. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x,\ on\ \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **4**